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Research Report CCS 481

EMPIRICAL TESTS OF THE ASSUMPTIONS
UNDERLYING MODELS FOR
FOREIGN EXCHANGE RATES

by

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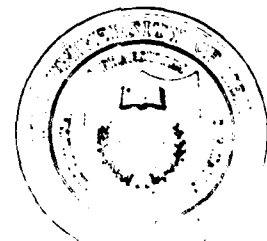
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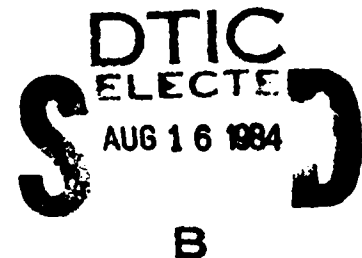CENTER FOR CYBERNETIC STUDIES

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ABSTRACT

By means of a very powerful statistical technique the basic linear stochastic process assumption of all existing intertemporal models for weak form efficiency in foreign exchange markets is rejected. Other foreign exchange models based on spot-forward and risk premium relationship are thereby also rejected. The tests were applied to the U.S. dollar vs. the Yen currency exchange market. Conclusions from the rejected models are thereby invalidated. Additionally, previous statistical forecast inference is to be suspected since forecast errors were found to be emphatically non-normal and nonlinear.

KEY WORDS

Accession For	
NTIS GRA&I	✓
DTIC TAB	☐
Unannounced	☐
Justification	
By	
Distribution/	
Availability Codes	
Dist	Avail and/or Special
A-1	

Market efficiency

Spot-forward prices

Stochastic (Gaussian and linear processes)

Stationary time series

Spectrum

Bispectrum

1. INTRODUCTION

In this paper we use a powerful, newly developed statistical technique to reject the basic assumption of linear stochastic process which underlies most models used in the foreign exchange literature, and in particular, all of the existing intertemporal models for weak form efficiency in the foreign exchange market. Weak form efficiency in the foreign exchange market has previously been tested with regressive, autoregressive, and autoregressive moving average models. This paper questions the appropriateness of those model forms for testing market efficiency. Thus the issue addressed herein is not one of market efficiency per se, but rather the correctness of the underlying assumptions of the models previously used to test market efficiency. Similarly, many models in foreign exchange which attempt to show the spot-forward, and risk premium relationship are questioned.

Tests are applied to one of the most closely watched currency exchanges; the U.S. dollar vs. Yen, and the results emphatically demonstrate that these spot and forward rates do not follow a linear stochastic process (and hence we reject all the regression and autoregressive, and autoregressive moving average models used in the literature). Since the underlying models postulated in these papers are wrong, the conclusions drawn previously about efficiency, risk premiums, exchange rate bias, and so on, in the foreign exchange market are severely challenged. It follows that efficiency in foreign exchange markets is a yet undecided issue. If markets are efficient, then our results show the price stochastic process must be nonlinear and non-Gaussian and efficient. Accordingly, henceforth intertemporal models

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for testing efficiency in the foreign exchange market must be nonlinear, non-Gaussian stochastic models. Additionally, forecast errors using the nominal rates and log rates are emphatically non-normal and nonlinear in both the additive and multiplicative error models so previous statistical inference is suspect.

The technique we use is a newly developed statistical test for linearity and Gaussianity of stationary time series due to Hinich (1982). By considering the bispectrum of the time series we are able to conclude that the series of spot, forward, and forecast errors are not compatible with either linear or Gaussian time series. These test are generally applicable to all time series, and are not particular to foreign exchange models alone. Consequently, these tests should be of interest to a wide audience of researchers doing empirical modeling and testing.

In section II we present some background definitions on time series models. Section III gives a brief review of some techniques currently used to model the relationship between the spot price, forward price, and forecast error in a weakly efficient market. Mostly these are regression or autoregressive moving average type models which are special cases of linear processes.[1] Section IV gives a brief motivation and discussion of the Hinich time series tests. Section V gives the results of applying these tests to the U.S. dollar to Japanese Yen foreign exchange rates. Conclusions and discussion is given in section VI.

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[1]The previous authors cannot be faulted for postulating linear models. We also find the pertinent series are serially uncorrelated so they do indeed masquerade as random walks. Actual statistical tests for linearity and Gaussianity, and the computational power to implement them are a very recent development in the time series literature. See Hinich (1982) for details.

The reader is again cautioned to remember that the focus of this paper is on the appropriateness of the models used in foreign exchange testing rather than whether or not the foreign exchange market is efficient. The presence or absence of efficiency, and model appropriateness are separate issues. However before efficiency can be validly addressed, the appropriate model must be applied. Therefore, this paper does not focus extensively on the literature addressing market efficiency, but rather on the modeling techniques most commonly used.

II. Linear and Gaussian models

We shall begin by giving some pertinent definitions and notation useful in the sequel for explaining our results.

A stationary time series $\{X(1), X(2), \ldots\}$ is a sequence of random variables such that the joint distribution of $X(k_1), \ldots, X(k_n)$ depends only on the difference between the k_i's, and not their precise values. Assuming the existence of moments, stationarity implies the mean $\mu = E[X(k)]$, the covariance $C_x(m) = E[X(n+m)X(n)] - \mu^2$ and third moments $E[X(n+r)X(n+s)X(n)]$ are independent of n. All joint moments will be stationary as well. To simplify notation from here on we shall center our series and assume $\mu = 0$.

If $\{X(1), X(2), \ldots\}$ are mutually independent, then the time series is called purely random. If $C_x(m) = 0$ for all $m \neq 0$, then the series is called

white noise. Purely random series are white, but not necessarily conversely. If the joint distribution of $\{X(k_1), \ldots, X(k_n)\}$ is multivariate normal, then the time series is called a Gaussian process. White Gaussian processes are purely random, but in general whiteness of a series does not imply the series is purely random. This is an important distinction since there are commonly used time series techniques (e.g. ARIMA models) which stop fitting the series model when the residuals appear to be white noise. Often researchers will then make the assumption that the residual series is Gaussian for convenience, and then accept this normality as a fact when doing hypothesis tests. The distinction is even confused in some leading textbooks. If the series is non-Gaussian, this may lead to extremely erroneous inferences.

A linear process is a time series which can be expressed in the form

$$(2.1) \qquad X(n) = \sum_{m=-\infty}^{\infty} a(m) \varepsilon(n-m)$$

where $\{\varepsilon(n)\}$ is a purely random series. This model includes all the autoregressive, autoregressive moving average models and certain martingale models so often used in finance and economics. If the series $\{\varepsilon(n)\}$ is Gaussian, then the original process $\{X(n)\}$ is also Gaussian. The converse is also true; $X(n)$ Gaussian implies $\varepsilon(n)$ Gaussian. In this paper, whenever we speak of accepting or rejecting linearity, we shall mean accepting or rejecting the stochastic model form (2.1).

With the preceeding definitions set in mind, we may now proceed to examine the models used to test and model market efficiency in particular, and the spot-forward rate relationship in general in foreign exchange models.

III. Models of the Spot-Forward rate relationship

There have been many articles which examine the relationship between the spot price, and the corresponding forward price in the foreign exchange market. Some authors attempt to formalize the notion of market efficiency by postulating a particular linear (often regression) relationship between the spot and forward price. The residual error terms, or sometimes the forecast errors are assumed to be independent identically distributed normal variates (white Gaussian noise) and hypothesis tests about the values of certain parameters in the model are used to infer market efficiency, exchange rate bias, risk premiums or other characteristics of interest. The use of linear models with normally distributed errors is made for statistical convenience. There is nothing in the notion of "efficiency", or in economic principles, which would necessarily dictate a linear processes with Gaussian white noise residual errors. Markets could be nonlinear and efficient. Previous researchers did not have access to newly developed statistical tests for

linearity and Gaussianity of time series, and so they used the well developed

theory based upon linear Gaussian processes without the ability to check if

their data were compatible with this assumption. We check this assumption in

Section 5.

In this section we shall briefly detail a few of the existing techniques

and models used for the foreign exchange market. More detailed summaries of

the empirical literature can be found in Kohlhagen (1978) and Levich (1979).

In section 5 we shall show that the underlying basic linear process

assumption involved in these models is wrong.

To formulate the models in question, we first introduce the following

notation. Let $S(t)$ denote the spot price at time t, and $F(T,t)$ denote the

forward price at time t for a forward contract on the spot price T periods of

time in the future (at time t+T). The forward price $F(T,t)$ is an estimate of

$S(t+T)$, and the models in question involve analysis of the relationship

between the two stochastic processes $\{S(t)\}$ and $\{F(T,t)\}$.

Regression and autoregressive models

Levich (1979) mentions that the regression model is one of the most

commonly used models in the context of efficient foreign exchange modeling.

This is the technique used by Kaserman (1973), Bilson (1976), Bilson and

Levich (1977), Frenkel (1977, 1978) and Stockman (1978). Basically, the model

is given by

$$(3.1) \qquad S(t+T) = a + bF(T,t) + u_t$$

where u_t has a normal distribution with mean zero. The null hypothesis is

that a = 0 and b = 1. If this statistical hypothesis is not rejected the

analyst concludes that the forward rate is an unbiased predictor. Levich
indicates that in many studies this fact is taken as a proof of efficient
markets, however the conclusions drawn in previous papers is not important to
us in this paper. Here we are primarily interested in the validity of the
equations put forth, for whatever purposes. Levich also quite properly notes
that such test are actually joint tests; vis a vis, the model being correct
and the market being efficient.

Cornell (1977), who also noted that all tests of efficiency are joint
tests, used an extension of the basic regression model in the form:

(3.2) $F(1, t-1) - S(t) = a_0 + a_1(S(t-1) - S(t-2))$, and
$$F(1, t-1) - S(t) = a_0 + a_1(S(t-1) - S(t-2)) + a_2(S(t-3) - S(t-4))$$

His findings led him to the conclusion that "the stochastic process
generating exchange rates changes can be characterized as the sum of the
constant drift term and random noise". This conclusion was rejected
statistically in Taylor (1980) in favor of a trend model with non-constant
drift.

Taylor ties the "weak" form of efficient markets with the general random
walk model as follows. In times of inflation, when prices have an upward
drift, the positive random walk is modelled by:

(3.3) $X_t = \mu_t + e_t$

where,

$\mu_t \geq 0$ and

$E(e_t) = 0$, $E(e_t e_{t+i}) = 0$ $(i \neq 0)$, $\text{cov}(\mu_s, e_t) = 0$ (all s,t)

Here X_t is the random walk process, μ_t is the drift series and e_t is the white-noise series. In the case of efficient markets we can describe μ_t by:

(3.4) $\qquad \mu_t = RF_t + RP_t; \qquad RP_t \geq 0$

where RF_t are returns from risk free investments and RP_t are risk premium series. Levich (1979) also concurs with the above rejection of the constant drift model. He remarks that "movement in the spot rate is likely to be dominated by a trend". According to our section 5 results, these trends must be nonlinear and non-Gaussian. Returning to the regression model (3.1), Frenkel (1976) examines the log of the rates in a manner analogous to those previously described:

(3.5) $\qquad \log S(t) = a + b \log F(1,t-1) + u_t$
$\qquad\qquad$ where u_t is Gaussian white noise

He checks the assumption that $a = 0$, $b = 1$ and u_t is serially uncorrelated. Since his null hypothesis is not rejected, he goes further into interpreting the efficient market implications. He narrows his examination to the question whether $F(1,t-1)$ indeed holds all the relevant information needed to determine $S(t)$ as it would in an efficient market. For that he tests (like Cornell (1977)) using the postulated linear model:

(3.6) $\qquad \log S(t) = a_0 + a_1 \log F(1,t-1) + a_2 \log F(2,t-2)$

He reports that for his set of data $F(2,t-2)$ has not added a significant explanation to the regression.

$\qquad$ Grauer, Lizenberger and Stehle (1976) suggest that the differences

between the forward rate and the expected value of the future spot rate

should be attributed to a systematic risk which is associated with the

foreign exchange position. They postulate the model

(3.7) $F(1,t-1) = E_{t-1}[S(t)] + RP(t)$

Where E_{t-1} is the expected value operator at time t-1 and RP(t) is the risk

premium (which depends on the systematic risk.) The efficient market

assumption enters the model via Samuelson's (1965) martingale argument. They

assume $E_{t-1}[S(t)] = S(t) + u(t)$ where u(t) is Gaussian white noise, and arrive

at the final equation

(3.8) $F(1,t-1) = S(t) + RP(t) + u(t).$

We note that if RP(t) in constant over time as some authors suggest, or if

RP(t) is a linear process itself(e.g. autoregressive), then F(1,t-1) - S(t)

is a linear process in the formulation (3.8).

Martinengo (1980) extends a model by Dornbusch (1976) in which market

equilibrium is formalized in terms of interest rates, level of prices, public

expenditure, money supply, full-employment income level, etc. The forward

rate is determined in this model by:

(3.9) $F(1,t-1) = \rho E_{t-1} S(t) + (1-\rho)E_{t-2} S(t)$

$0 \leq \rho \leq 1$

i.e., the forward rate is formed in an adaptive way by the sequence of

expectations on the future spot rate. The martingale argument used in

deriving (3.8) now applies to obtain a linear model in this situation as

well.

Properties of the Forecast Error

In various researches the effort was focused on the analysis and interpretation of the forecast error. Studies following this approach include Aliber (1974), Kohlhagen (1974, 1975), Giddy and Dufey (1975) and Levich (1977). Under the hypothesis of an efficient market the forward price should be the best estimate of the future spot price so the authors conclude the forecast error:

$$(3.10) \quad P \epsilon_A(t) = S(t) - F(1,t-1)$$

should be purely randomly noise. The subscript A has been introduced here to designate the fact that this is an additive error model, and $P\epsilon$ stands for price forecast error. Although not considered explicitly by other authors, there are several alternative forecast error models which need investigation relative to efficient market modeling. The first is the multiplicative model for price forecast error

$$(3.11) \quad P \epsilon_M(t) = S(t)/F(1,t-1).$$

Other contending models involve rates (log prices) rather than the prices themselves. By analogy to (3.5) we have the additive error model for rate forecast errors $(R\epsilon)$

$$(3.12) \quad R \epsilon_A(t) = \ln S(t) - \ln F(1,t-1) = \ln[S(t)/F(1,t-1].$$

Of course $R \epsilon_A(t) = \log P \epsilon_M(t)$ so modeling a linear (random walk) relationship for rates is equivalent to a log linear (geometric random walk) relationship for prices.

The final model which needs to be examined in the forecast error context is the multiplicative model for rate forecast errors:

(3.13) $R_{\epsilon_M}(t) = \ln S(t)/\ln F(1,t-1)$.

The idea which all of the models (3.1) - (3.13) share is the postulated existence of a linear time series stochastic error term. Often for statistical validity these time series need to be assumed Gaussian as well. In the next section we show how one can test for linearity and Gaussianity of a time series model.

IV. Statistical Tests for linearity and Gaussianity of Time series

Subba Rao and Gabr (1980), and Hinich (1982), present statistical tests for determining whether a given stationary time series $\{X(n)\}$ is linear (i.e. has the form (2.1)) and Gaussian. It is possible that $\{X(n)\}$ is linear without being Gaussian, but all the stationary Gaussian time series are linear.

Both the Subba Rao and Gabr, and Hinich tests are based upon the sample bispectrum of the time series. The Hinich test is non parametric, and is robust. Additionally the Hinich test is conservative in the presence of nonstationarity of the time series (a frequent occurence in finance), so if we can reject linearity and/or Gaussianity using the Hinich test, the rejection would continue even if the series was non stationary. Accordingly the tests presented in this paper use the Hinich test.

Let $\{X(n)\}$ be a stationary time series and assume without loss of generality that $E[X(n)] = 0$. The spectrum of $\{X(n)\}$ is the Fourier transform of the autocovariance function $C_x(n) = E[X(t+n)X(t)]$;

$$S(f) = \sum_{n=0}^{\infty} C_x(n)\exp\{-2\pi i f n\} \, .$$

Many papers in finance and economics use the spectrum $S(f)$ as a way to examine the correlation structure of $X(n)$. See Granger and Morgenstern (1963) for numerous applications of spectral analysis techniques to finance. In particular, $X(n)$ is serially uncorrelated (white noise) if $S(f)$ is constant.

The bispectrum of $\{X(n)\}$ is defined to be the (two dimensional) Fourier transform of the third moment function $C_{xx}(n,m) = E[X(t+n)X(t+m)X(t)]$,

$$B(f_1,f_2) = \sum_m \sum_n C_{xx}(n,m)\exp\{-2\pi i f_1 n - 2\pi i f_2 m\}.$$

A rigorous introduction to the bispectra and its symmetries and properties can be found in Brillinger and Rosenblatt (1967). For our purposes, the important thing about the bispectrum is it allows a statistical test for linearity and Gaussianity of a time series.

Suppose $X(n)$ is a linear time series, i.e. has the form (2.1). Then it can be shown that the spectrum of $\{X(n)\}$ is of the form

$$(4.1) \qquad S(f) = \sigma_\epsilon^2 |A(f)|^2$$

and the bispectrum of $\{X(n)\}$ is of the form

$$(4.2) \qquad B(f_1,f_2) = A(f_1)A(f_2)A^*(f_1+f_2)\mu_3$$

where $\mu_3 = E\epsilon^3(t)$, $A(f)$ is the transform of the coefficient series,

$$A(f) = \sum_{n=0}^{\infty} a(n)\exp(-2\pi i f n)$$

and A^* is the complex conjugate of A.

From (4.1) and (4.2) it follows that

$$(4.3) \qquad \frac{|B(f_1,f_2)|^2}{S(f_1)S(f_2)S(f_1+f_2)} = \frac{\mu_3^2}{\sigma_\epsilon^6}$$

is constant over all frequency pairs (f_1, f_2) if $\{X(n)\}$ is linear. Moreover, since $\mu_3 = 0$ for the normal distribution, the above constant is zero if the time series is Gaussian.

The relationship (4.3) is the basis of the Hinich tests. Constructing an estimate of the bispectrum $\hat{B}(f_1,f_2)$, and of the spectrum $\hat{S}(f)$, he estimates the ratio in (4.3) at different frequency pairs (f_1,f_2) by $|\hat{B}(f_1,f_2)|^2/\hat{S}(f_1)\hat{S}(f_2)\hat{S}(f_1+f_2)$. If these ratios differ too greatly over different frequency pairs, he rejects the constancy of the ratio, and hence linearity of the time series $\{X(n)\}$. If the estimates differ too greatly from zero, he rejects the Gaussianity time series model. The constant $\mu_3^2/\sigma_\epsilon^6$ is the square of Fisher's skewness measure for the ϵ series.

The test statistic he derives for testing linearity is based upon the inner quartile range of the estimated ratio over the set of pertinent frequency pairs. If the ratio in (4.3) is constant, then the inner quartile range is small. If it is not constant, then the inner quartile range is larger. This test is robust and fairly powerful at sample sizes as low as 256. (c.f. Ashley and Hinich (1983)). It is also conservative with respect to non stationarity since non stationarity would tend to smear the peaks in the estimated bispectrum, and hence reduce the dispersion of the estimated ratio. The test statistic for linearity is asymptotically normal so

significance is readily determined from standard normal tables. See Hinich

(1982) for the precise formulae and proofs concerning this test for

linearity.

The test for Gaussianity of the time series involves testing for the

ratio (4.3) being zero. Hinich (1982) derives an asymptotically normal test

statistic based upon the estimated ratio (4.3) in this situation as well.

It should be emphasized that the time series under study can be serially

uncorrelated and still fail to be either linear or Gaussian. Indeed in an

efficient market one might expect such a result since such series notoriously

mascarade as white noise series.

In the next section we shall show the results of applying the Hinich

tests to foreign exchange data.

V. Lack of Linearity and Gaussianity of Foreign Exchange Data

In this section we shall present the results of implementing the

preceeding statistical tests to the analysis of forward, spot, and forecast

errors in both the original price quote form and also in the log price (rate)

form. We have chosen for analysis the U.S. dollar to Japanese Yen exchange

rates since this is ostensibly one of the most closely watched and tightly

arbitraged currency exchanges. If linearity and/or Gaussianity is to be

found in foreign exchange rates, this is a likely place to find it. We

examine two time periods, from January 1, 1981 - mid 1982 and from December 12, 1981 to mid 1983.[2] We have used daily quotes for rates taken from The Wall Street Journal, using the thirty day forward rate $F(30,t)$ and the corresponding spot rate $S(30+t)$.

In all cases the spectrum of the series analyzed is virtually flat, indicating a close approximation to serially uncorrelated noise. This is consistent with these rates mascarading as random walk processes. Table 1 shows the results of the analysis applied to the spot, log spot, forward, and log forward time series. The Hinich tests yield standard normal variates for the test statistic if the hypothesized time series model is indeed true. A 1% level of significance is an entry of 2.57, and the results shown in Table 1 are significant at almost any significance level. Overall the message of Table 1 is clear. The series are nonlinear and non-Gaussian.

Perhaps more surprising from a conceptual point of view are the results of Table 2 concerning the nonlinearity and non-Gaussianity of the various forecast error models. Nonlinearity might be present in the spot price for example, but one would hope that the forward prices would incorporate this nonlinearity also, so that the difference would be Gaussian white noise. This is emphatically not so. All the forecast error models also reject linearity and Gaussianity. Some implications of this rejection are given in the concluding section.

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2. We have deliberately introduced an overlapping of the time periods to increase the robustness of the results.

TABLE 1

RESULTS OF TEST FOR FOREIGN EXCHANGE RATE TIME SERIES LINEARITY AND GAUSSIANITY.

Entries are standard normal variates if the model is correct

		1/2/81 – Mid 82	12/31/81 – Mid 83
Spot Price Process S(t)	Linearity test Statistic	232.749	17.699
	Gaussianity test Statistic	104.3915	20.736
30 day Forward Price Process F(30,t)	Linearity test Statistic	350.266	25.418
	Gaussianity test Statistic	119.5641	28.0713
log spot Price Process ln S(t)	Linearity test Statistic	9.979	2.7039
	Gaussianity test Statistic	25.7844	6.6853
log Forward Process ln F(30,t)	Linearity test Statistic	948.82	0.7995
	Gaussianity test Statistic	164.6701	3.9083

TABLE 2

RESULTS OF TESTS FOR FORECAST ERROR TIME SERIES LINEARITY AND GAUSSIANITY

		1/2/81 – Mid 82	12/31/81 – Mid 83
Forecast error Process S(t+30)–F(30,t)	Linearity test Statistic	186.39	461.134
	Gaussianity test Statistic	134.2781	129.0403
Forecast error Process ln S(t+30)–lnF(30,t)	Linearity test Statistic	115.14	356.60
	Gaussianity test Statistic	104.3095	106.7738
Forecast error Process S(t+30)/F(30,t)	Linearity test Statistic	277.025	239.658
	Gaussianity test Statistic	217.5779	204.594
Forecast error Process lnS(t+30)/lnF(30,t)	Linearity test Statistic	77.738	190.434
	Gaussianity test Statistic	271.9302	270.0571

All numbers should be compared with standard normal table for significant levels. For comparison purposes, a Z-value of 20 corresponds to a p-value of approximately 10^{-48}. A Z value of 29 corresponds approximately to a p value of 10^{-140}. The life of the universe is approximately 10^{100} days.

VI. Conclusion and Discussion

Tables 1 and 2 exhibit the results of tests for linearity and
Gaussianity of models which include important cases of the models (3.1) -
(3.13). It can be seen that both linearity and Gaussianity are emphatically
rejected at virtually all levels of significance. (The exception being the
log forward sequence over the time span 12/12/81 to mid 83. It is possible
that this process was linear but non-Gaussian over this time period).
Moreover, since the Hinich tests are conservative in the presence of
nonstationarity these dramatic rejections of previous models cannot be
attributed to nonstationarity of the series in question. These series are
not linear or Gaussian. Since the tests of efficiency of the foreign exchange
market are based upon a model which is fundamentally incorrect (namely linear
and Gaussian), the conclusions of the previous studies must be reexamined.
The inference concerning significance of the parameter values in equations
(3.1), (3.2). (3.5), and (3.6) is based on a normal distribution of
residuals, and the sampling distributions in the non-Gaussian case is not
known. Thus, previous statistical analysis must also be suspect. In a sense
our result is similar to Roll's (1977) critique of tests of the capital asset
pricing model; since the fundamental models used are rejectable, the
conclusions have limited validity. This is not to say that the exchange
market is not efficient. We have not tested efficiency. The market may
indeed by efficient, but it will require nonlinear, non-Gaussian stochastic
process models to be able to provide a nonrefutable models for statistical
testing. A review of pertinent nonlinear time series models is given in
Priestley (1980).

An interesting conclusion concerning the model (3.8) should also be noted. Either the residual error $u(t)$ is a nonlinear, non-Guassian process, or else the risk premium $RP(t)$ is nonlinear and non-Gaussian (or both $u(t)$ and $RP(t)$ are). If we believe in a constant or linear (say autoregressive) risk premium, then inference must involve nonlinear, non-Gaussian residuals. This is since the forecast error $F(1,t-1) - S(t)$ is nonlinear and non-Gaussian. Again, possible nonstationarity only makes the case stronger for nonlinear modeling.

As a final note, it should be mentioned that it is to be expected that many of the conclusions and results presented in this paper for foreign exchange markets will carry over to other markets in which efficiency is linearily modeled. This will be investigated in subsequent papers, as well as certain nonlinear time series models.

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

By means of a very powerful statistical technique the basic linear stochastic process assumption of <u>all</u> existing intertemporal models for weak form efficiency in foreign exchange markets is rejected. Other foreign exchange models based on spot-forward and risk premium relationship are thereby also rejected. The tests were applied to the U.S. dollar vs. the Yen currency exchange market. Conclusions from the rejected models are thereby invalidated. Additionally, previous statistical forecast

20. Abstract (continued)
    inference is to be suspected since forecast errors were found to be
    emphatically non-normal and nonlinear.

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